


Benha University Faculty of Engineering – Shoubra Department of Energy and Sustainable Energy Course: Mathematics 4 Code: EMP 202		Final Exam Date: June 10, 2015 Duration: 3 hours Answer All questions
The exam consists of one page	No. of questions: 4 Total Mark: 40	
<p>Question 1</p> <p>Solve the following equations:</p> <p>(a) $y \sin x \, dx + (1 + y^2)dy = 0$ (b) $(y + x \cos y)dy + (x + \sin y)dx = 0$ (c) $y'' - 2y' - 3y = e^{-x} + 3^x$ (d) $y'' + y' = 3 + x^2$ (e) $(D^2 + 4)y = 1 + \cos 2x$ (f) $(D^2 + 1)y = \tan x$</p> <p>Question 2</p> <p>(a) Prove that: If $f(t)$ is function with Laplace transformation $F(s)$. Then</p> $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s) \, ds$ <p>(b) Find Laplace transformation of the following:</p> <p>(i) $f(t) = 3 + t^3 + \sinh 3t$ (ii) $f(t) = \sin 3t + t \cos t$ (iii) $f(t) = \sin(t - 3), t > 3$ (iv) $f(t) = \frac{e^{3t} - e^{2t}}{t}$</p> <p>(c) Find the inverse Laplace transform of: (i) $F(s) = \frac{s}{s^2 - 2s - 3}$ (ii) $F(s) = \tan^{-1}(s - 2)$ (d) Solve the equation: $y'' + 2y' + y = e^{-t}, y(0) = 0, y'(0) = 1$.</p> <p>Question 3</p> <p>(a) Find the root of the equation $x^3 + 3x - 10 = 0$ by using Newton method. (b) Construct the difference table to the following data.</p> <p>Hence find interpolation polynomial interpolate the function $y = f(x)$ at the points $(0, 1), (0.1, 1.32), (0.2, 1.68), (0.3, 2.08), (0.4, 2.52)$.</p> <p>Question 4</p> <p>(a) Approximate the integrals $\int_0^1 \sqrt{1+x} \, dx$ using Sampson's rule. Estimate the error by computing the exact value. (b) Solve the differential equation : $y' = x + y, 0 < x < 1, y(0) = 1$ using Euler method considering $h = 0.2$. Estimate the error at $x = 0.4$ by comparing your result.</p>	<p>9</p> <p>2</p> <p>4</p> <p>2</p> <p>3</p> <p>10</p> <p>10</p>	

Good Luck

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Answer

Question 1

(a) $y \sin x \, dx + (1 + y^2)dy = 0$

Divide by y , we get : $\sin x \, dx + \left(\frac{1}{y} + y\right)dy = 0$

Then, the solution is : $-\cos x + \ln y + \frac{1}{2}y^2 = c$

(b) $(y + x \cos y)dy + (x + \sin y)dx = 0$

Since $P = x + \sin y$, $q = y + x \cos y$ and $P_y = \cos y = q_x$. The equation is exact.

Then $\int (x + \sin y)dx = \frac{1}{2}x^2 + x \sin y$, $\int (y + x \cos y)dy = \frac{1}{2}y^2 + x \sin y$

Then, the solution is : $\frac{1}{2}x^2 + \frac{1}{2}y^2 + x \sin y = c$

(c) $y'' - 2y' - 3y = e^{-x} + 3^x$

The A.E is : $k^2 - 2k - 3 = (k - 3)(k + 1) = 0$. Then $k = 3, -1$.

Then $y_c = Ae^{3x} + Be^{-x}$

$$y_p = \frac{1}{(D-3)(D+1)}e^{-x} + \frac{1}{D^2-2D-3}e^{(\ln 3)x} = \frac{1}{-4}x \cdot e^{-x} + \frac{1}{(\ln 3)^2 - 2 \ln 3 - 3}e^{(\ln 3)x}$$

$$y = y_c + y_p$$

(d) $y'' + y' = 3 + x^2$

The A.E is : $k^2 + k = k(k + 1) = 0$. Then $k = 0, -1$.

Then $y_c = A + Be^{-x}$

$$\begin{aligned} y_p &= \frac{1}{D(D+1)}(3 + x^2) = \frac{1}{D}(1 + D)^{-1}(3 + x^2) = \frac{1}{D}(1 - D + D^2 + \dots)(3 + x^2) \\ &= \frac{1}{D}(3 + x^2 - 2x + 2) = 5x + \frac{1}{3}x^3 - x^2 \end{aligned}$$

$$y = y_c + y_p$$

(e) $(D^2 + 4)y = 1 + \cos 2x$

The A.E is : $k^2 + 4 = 0$. Then $k = \pm 2i$.

Then $y_c = A \cos 2x + B \sin 2x$

$$y_p = \frac{1}{D^2+4}(1 + \cos 2x) = \frac{1}{D^2+4}1 + \frac{1}{D^2+4}\cos 2x = \frac{1}{4} + \frac{x}{4}\sin 2x$$

$$y = y_c + y_p$$

(f) $(D^2 + 1)y = \tan x$

The A.E is : $k^2 + 1 = 0$. Then $k = \pm i$.

Then $y_c = A \cos x + B \sin x$

$$w = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$w_1 = \begin{vmatrix} 0 & \sin x \\ 1 & \cos x \end{vmatrix} = -\sin x$$

$$w_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & 1 \end{vmatrix} = \cos x$$

Then $y_p = \cos x \int \frac{w_1}{w} \tan x \, dx + \sin x \int \frac{w_2}{w} \tan x \, dx$

$$= \cos x \int \frac{-\sin x}{1} \tan x \, dx + \sin x \int \frac{\cos x}{1} \tan x \, dx$$

$$= \cos x \int (\cos x - \sec x) \, dx + \sin x \int \sin x \, dx$$

$$= \cos x (\sin x - \ln(\sec x + \tan x)) + \sin x (-\cos x)$$

$y = y_c + y_p$

.....(9 Marks)

Question 2

(a)Theorem

.....(2 Marks)

(b)(i) $f(t) = 3 + t^3 + \sinh 3t$

$$F(s) = \frac{3}{s} + \frac{3!}{s^4} + \frac{3}{s^2-9}$$

(ii) $f(t) = \sin 3t + t \cos t$

$$F(s) = \frac{3}{s^2+9} - \left(\frac{s}{s^2+1}\right)' = \frac{3}{s^2+9} - \frac{1-s^2}{(s^2+1)^2}$$

(iii) $f(t) = \sin(t - 3), t > 3$

Since $L\{\sin t\} = \frac{1}{s^2+1}$. Then $F(s) = \frac{1}{s^2+1} e^{-3s}$

(iv) $f(t) = \frac{e^{3t}-e^{2t}}{t}$

Since $L\{e^{3t} - e^{2t}\} = \frac{1}{s-3} - \frac{1}{s-2}$. Then $F(s) = \int_s^\infty \frac{1}{s-3} - \frac{1}{s-2} \, ds = \ln \frac{s-2}{s-3}$

.....(4 Marks)

$$(c)(i) F(s) = \frac{s}{s^2 - 2s - 3}$$

By method of partial fractions, we get $F(s) = \frac{3/4}{s-3} + \frac{1/4}{s+1}$

$$\text{Then } f(t) = \frac{3}{4}e^{3t} + \frac{1}{4}e^{-t}$$

$$(ii) F(s) = \tan^{-1}(s - 2)$$

Since $F'(s) = \frac{1}{(s-2)^2 + 1}$ and $L^{-1}\left\{\frac{1}{(s-2)^2 + 1}\right\} = e^{2t} \sin t = -t f(t)$

$$\text{Then } f(t) = -\frac{e^{3t} \sin t}{t}$$

.....(2 Marks)

$$(d) y'' + 2y' + y = e^{-t}$$

$$L\{y'' + 2y' + y\} = \{e^{-t}\}$$

$$\text{Then } s^2Y - sy(0) - y'(0) + 2(sY - y(0)) + Y = \frac{1}{s+1}$$

From the initial conditions, we get

$$s^2Y - 1 + 2sY + Y = \frac{1}{s+1} \quad \text{or} \quad Y = \frac{s+2}{(s+1)^3} = \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}$$

Then, the solution is : Then $y(t) = t.e^{-t} + \frac{t^2}{2}e^{-t}$

.....(3 Marks)

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Question 3 10 Marks

(a) Find the root of the equation $x^3 + 3x - 10 = 0$ by using Newton method.

Answer

Newton's method

$$x^3 + 3x - 10 = 0$$

$$f(x) = x^3 + 3x - 10$$

$$f'(x) = 3x^2 + 3$$

$$x_{n+1} = x_n - \frac{x_n^3 + 3x_n - 10}{3x_n^2 + 3} = \frac{2x_n^3 + 10}{3x_n^2 + 3}$$

i	x_i	x_{i+1}	$f(x_{i+1})$
0	1	2	4

1	2	1.733333333	0.407703704
2	1.733333333	1.699395733	0.005950068
3	1.699395733	1.698885604	1.32658E-06
4	1.698885604	1.69888549	6.39488E-14
5	1.69888549	1.69888549	0

$R = 1.69888549$

(b) Construct the difference table to the following data.

Hence find interpolation polynomial interpolate the function

$y = f(x)$ at these points (0, 1), (0.1, 1.32), (0.2, 1.68), (0.3, 2.08), (0.4, 2.52).

Answer

The difference table

x	y	∂f	$\partial^2 f$	$\partial^3 f$	$\partial^4 f$
0	1	0.32	0.02	0.04	- 0.06
0.1	1.32				
		0.34	0.06	- 0.02	
0.2	1.68				
		0.4	0.04		
0.3	2.08				
		0.44			
0.4	2.52				

$$\begin{aligned}
P_3(x) &= y_0 + \partial f(x - x_0) + \partial^2 f(x - x_0)(x - x_1) \\
&\quad + \partial^3 f(x - x_0)(x - x_1)(x - x_2) + \partial f(x - x_0)(x - x_1)(x - x_2)(x - x_3) \\
&= 1 + 0.32(x - 0) + 0.02(x - 0)(x - 0.1) + 0.04(x - 0)(x - 0.1)(x - 0.2) \\
&\quad - 0.06(x - 0)(x - 0.1)(x - 0.2)(x - 0.3)
\end{aligned}$$

Question (4) 10 Marks

(b) Approximate the integrals $\int_0^1 \sqrt{1+x} dx$ using Sampson's rule. Estimate the error by computing the exact value.

Answer

$$\int_0^1 \sqrt{1+x} dx$$

Solution: (a) by Trapezoidal Rule

i	x_i	$f(x_i)$	$2f(x_i)$
0	0	1	1
1	0.1	1.048808848	2.097617696
2	0.2	1.095445115	2.19089023
3	0.3	1.140175425	2.28035085
4	0.4	1.183215957	2.366431913
5	0.5	1.224744871	2.449489743
6	0.6	1.264911064	2.529822128
7	0.7	1.303840481	2.607680962
8	0.8	1.341640786	2.683281573
9	0.9	1.378404875	2.75680975
10	1	1.414213562	1.414213562

$$I = 1.21882942$$

The exact value $\int_0^1 \sqrt{1+x} dx = \frac{2}{3} \left[(1+x)^{3/2} \right]_0^1 = \frac{2}{3} \left[2^{3/2} - 1 \right] = 1.21895141649764$

Error=1.22E-4

(b) Solve the differential equation $y' = x + y$, $0 < x < 1$, $y(0) = 1$ using Euler method considering $h = 0.2$. Estimate the error at $x = 0.4$ by comparing your result with the exact solution of the problem.

Answer

$$y' = (x + y), \quad 0 < x < 1, \quad y(0) = 1$$

Answer:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$\begin{aligned} y_{n+1} &= y_n + hf(x_n, y_n) \\ &= y_n + 0.2[x_n + y_n] \\ &= y_n + 0.2[0.2n + y_n] \end{aligned}$$

$$y_{n+1} = y_n + (0.2)^2 n + 0.2y_n$$

$$y_{n+1} = 1.2y_n + (0.2)^2 n$$

$$\boxed{n = 0, 1, 2, 3, 4, \dots}$$

$$y_1 = 1.2y_0 + (0.04)(0) = 1.2(1) + 0 = 1.2$$

$$y_2 = 1.2y_1 + (0.04)(1) = 1.2(1.2) + (0.04)(1) = 1.48$$

$$y_3 = 1.2y_2 + (0.04)(2) = 1.2(1.48) + (0.04)(2) = 1.856$$

$$y_4 = 1.2y_3 + (0.04)(3) = 1.2(1.856) + (0.04)(3) = 2.3472$$

$$y_5 = 1.2y_4 + (0.04)(4) = 1.2(2.3472) + (0.04)(4) = 2.97664$$

Exact solution

$$y' = (x + y)$$

$$y' - y = x$$

$$e^{-x} y' - ye^{-x} = xe^{-x}$$

$$d(ye^{-x}) = xe^{-x}$$

$$ye^{-x} = \int xe^{-x} dx = -xe^{-x} - e^{-x} + c$$

$$ye^{-x} = -xe^{-x} - e^{-x} + c$$

Substitute by the initial condition

$$ye^{-x} = -xe^{-x} - e^{-x} + c$$

$$1 = 0 - 1 + c \Rightarrow c = 2$$

$$\therefore ye^{-x} = xe^{-x} - e^{-x} + 2$$

$$y = -x - 1 + 2e^x$$

$$X=0.4$$

$$y = -0.4 - 1 + 2e^{-0.4} = 1.583649395$$

$$\text{Error} = 1.48 - 1.583649395 = 0.103649395$$

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